Vibrating Wire Technique and Phase Lock Loop for Finding the Magnetic Axis of Quadrupoles

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***Abstract*— The tolerance in the alignment of the quadrupoles in the linacs and in the undulator lines of the Swiss Free Electron Laser (SwissFEL), the next project at the Paul Scherrer Institute (PSI), will be about 1 µm. This accuracy will be reached using beam alignment techniques. To minimize the commissioning time, it is also requested to install the quadrupoles with a precision of ±50 µm. To achieve this goal, the vibrating wire technique will be used in association with other systems to determine the position of the magnetic axis of the quadrupoles with respect to external fiducials. After a general introduction to the project and to the instrumentation principle, the novel approach developed at PSI is presented. The main innovation consists in the use of a phase lock loop (PLL) for maintaining the wire in its resonance condition. This approach simplifies the operation of the system in case the wire is moved or replaced and increases substantially the reliability of the measurement outcomes in the frame of a series test campaign.**

***Index Terms*—Magnetic measurements, single stretched wire, vibrating wire technique, magnetic axis determination, Phase Lock Loop.**

I. INTRODUCTION

THE alignment of magnets in a linac-based free electron laser is a challenge especially for the quadrupole magnets in the undulator section, where the electrons must overlap and resonate with the photon beam. In the SwissFEL [1], the next project at the Paul Scherrer Institute (PSI), the trajectory straightness shall be within a few micrometers [2] corresponding to an alignment accuracy of the magnets of about 1 µm. This target can be achieved only with a beam-based alignment procedure (BBA) [3]. The measurement of the magnetic axis is nevertheless required to implement an effective BBA. The initial alignment error must be minimised to implement a simple and fast BBA and for the investigation of the reproducibility of the magnetic axis after current cycles. The axis movements at different excitation levels have also to be carefully studied with dedicated measurements, the BBA being repeated in case the changes exceed the micrometer accuracy level. In the main linacs of the SwissFEL facility the alignment requirements are more relaxed and the errors should not exceed a few tens of micrometers. In this region of the accelerator the 200 quadrupole magnets are indeed not equipped with motorized stages and they must be physically realigned in case the error exceeds the corrector magnet capability.

The measurement of the magnetic axis consists of two phases: the detection of the minimum field strength in a local reference system by means of a magnetic sensor, and the transfer of the axis coordinates to an external reference, i.e. to the magnet frame. A review of the various types of magnetic sensors and survey systems with their associated errors and measurement uncertainty can be found in [4].

At PSI the vibrating wire technique was selected to detect the magnetic axis of the SwissFEL quadrupoles. A single conducting wire is stretched through the magnet aperture and driven by an alternating current. When the wire is off the magnetic axis it vibrates. The high sensitivity of this technique is based on the excitation of the natural resonance of the wire, which enhances the ratio between forces (magnetic field) and displacement. With this system both the horizontal and vertical offsets as well as pitch and yaw angles are easily accessible. This technique is very flexible and it can be easily implemented also for different magnets and aperture sizes.

In the following we report the first magnetic axis measurements of SwissFEL quadrupoles with the vibrating wire system bench assembled at PSI. The benefits of a phase lock loop (PLL) to set and keep the wire in resonance are presented for the specific PSI choice to move the wire and not the magnet in searching for the magnetic axis.

II. DESCRIPTION OF THE VIBRATING WIRE SYSTEM SETUP AT PSI

The single stretched wire used for quadrupole magnetic axis determination has been already described in the literature [5], [6] and only a few formulas for the scope of this paper will be recalled. The equation of motion in the horizontal *ZX*-plane of an AC driven stretched wire with mass per unit length (μ) in a vertical magnetic field $B_y(z)$ along the wire, is given by:

$$\mu \frac{\partial^2 x}{\partial t^2} = T \frac{\partial^2 x}{\partial z^2} - \gamma \frac{\partial x}{\partial t} + B_y(z) I_0 \cos \omega t \, , \qquad (1)$$

which represents forced vibration with damping.
$B_y(z)I_0\cos\omega t$ is the Lorentz (driving) force per unit length, γ is a damping factor, and T is the wire tension. Under the boundary condition $x(z=0) = x(z=L) = 0$ and the assumption that also the magnetic field is zero at the wire

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endpoints, the solution to the Eq. (1) is:

$$x(z,t) = \sum_n \frac{-I_0 B_{yn}}{\mu\sqrt{(\omega_n^2 - \omega^2)^2 + ((\gamma/\mu)\omega)^2}} \sin\left(\frac{n\pi z}{L}\right) \times \cos(\omega t + \varphi_n), \tag{2}$$

$$\varphi_n = \arctan\left(\frac{(\gamma/\mu)\omega}{\omega^2 - \omega_n^2}\right)$$

where $\omega_n = \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$ and $B_{yn} = \frac{2}{L}\int_0^L B_y(z)\sin\left(\frac{n\pi z}{L}\right)dz$.

Near resonance, when $\omega \approx \omega_n$, one term dominates the sum in the solution and therefore,

$$x(z,t) \approx \frac{-I_0 B_{yn}}{\gamma\omega_n} \sin\left(\frac{n\pi z}{L}\right) \cos(\omega_n t + \pi/2) \tag{3}$$

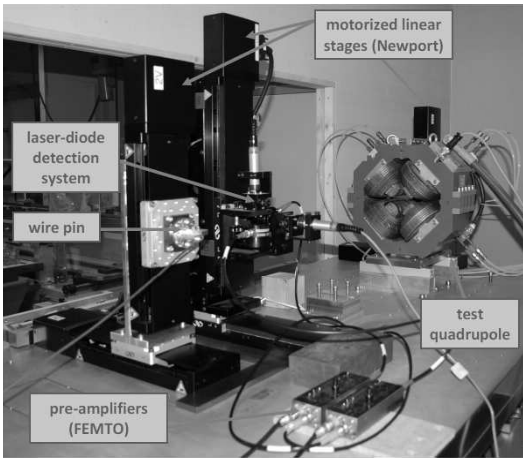

Fig. 1. The vibrating wire measurement system (test stand) at PSI.

The PSI measurement system is represented in Fig. 1. A copper-beryllium wire (0.125 mm diameter) is stretched and fixed between two pins at a distance *L* of 1.2 m. Each pin is mounted on two linear motorized stages for vertical and horizontal displacements. The magnet is fixed at 75% of the wire length. AC current is supplied to the wire to excite the second harmonic to find the vertical and horizontal offsets. The fourth harmonic is used to find the pitch and yaw angles of the magnetic axis with respect to the geometric axis [6]. When the wire reaches its target position (i.e. it is placed along the magnetic axis), the pin locations with respect to the magnet reference points are measured with a FARO© arm.

The wire vibration detector consists of two laser-photodiode pairs for horizontal and vertical vibration detection. They are as well mounted on two linear motorized stages for vertical and horizontal adjustment in accordance with the actual wire position. It is placed at 13% of the wire length where the amplitude of the vibration in the second as well as in the fourth harmonic can be detected, thus avoiding displacements of the detector along the wire between offsets and yaw/pitch measurements. The signals from the photodiodes are pre-amplified via two "FEMTO DLPCA-200" current-to-voltage amplifiers and demodulated at the current driving frequency with a lock-in amplifier (Zurich Instruments [7]).

III. IMPLEMENTATION OF PLL

PLL is a control system of the lock-in amplifier that keeps the phase difference of the input and output signal at a given value by adjusting the frequency of amplifier's oscillator that generates the output signal. The PLL control system of the Zurich Instruments lock-in amplifier is schematically represented in Fig. 2. The input voltage signal is proportional to the wire vibration amplitude, which in resonance condition has a 90 degree phase shift to the wire current, the output of the lock-in. The phase detector in the PLL compares the phase difference of these two signals and the error with the desired phase difference drives the PID controller elements which in turn drive the internal frequency oscillator of the lock-in, used as output to drive the wire.

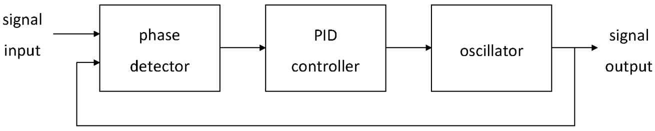

Fig. 2. Schematic overview of the PLL control system.

The vibration amplitude and phase w.r.t the wire current have been measured as a function of the driving frequency of the wire. Fig. 3 is the result of the measurement taken at 37 µTm quadrupole field integral. Due to the low damping of the system, there is a strong peak in vibration amplitude at resonance frequency and the curve of the phase is very steep around the resonance frequency. Therefore, any fluctuations or change in resonance frequency will introduce additional measurement errors or measurement points that are out of line.

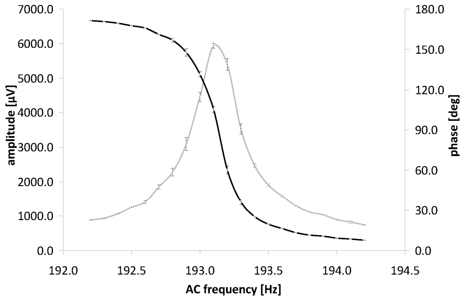

Fig. 3. Measured phase (black curve) and amplitude (grey curve) of wire vibration as a function of wire driving frequency. Resonance occurs at 193.2 Hz and 90 degree phase shift to the wire current.

The resonance frequency of the wire is very sensitive to ambient temperature. Sudden temperature changes from increasing or decreasing the wire current or slow temperature changes from the quadrupole heating will affect the resonance frequency. Resonance frequency changes of about 0.1 Hz/ºC have been observed. With this in mind, the advantages of the use of a PLL in the vibrating wire system are clear. With PLL, the wire will remain in resonance even if its resonance frequency is changed, for example due to small temperature fluctuations or other unforeseeable influences, or foreseeable ones, e.g. to wire movements. It was a specific choice by PSI to use a system in which the wire is moved instead of the quadrupole. The next sections describe measuring the magnetic axis using the moving wire (stationary magnet).

A. *Quadrupole axis offset measurements*

The effect of the PLL control system has been tested in the detection of the horizontal offset of the magnetic axis with respect to the geometric axis for a quadrupole with strength 0.36 T at 10 A and an effective length of 170 mm.

The amplitude and phase of horizontal vibration have been measured for different horizontal wire offsets and at each offset they were recorded 5 times over several seconds with and without use of PLL. The system characteristics and chosen lock-in parameters are given in Table I. The result of the average demodulated signal's amplitude and phase with the RMS error at each relative horizontal wire offset x-x_0 is shown in Fig. 4. The magnetic axis offset x_0 is defined by the intersection of two linear fits. As a figure of merit to define the error in finding the magnetic axis, the sum of the average measurement error σ_{meas} and error of the fit σ_{fit} is divided by the slope. This value will be defined as the resolution R and the slope of the fit will be defined as the sensitivity S of the system. The analysis summarized in Table II shows that the signal stability increases and the resolution improves when PLL is used to find the magnetic axis. The magnetic axis measurement has been repeated several times under different conditions (temperature changes and mechanical influences from the environment). Although the reproducibility in finding the magnetic axis in all measurements was higher with the PLL (0.7 µm versus 1.4 µm) both approaches are acceptable for the target of the SwissFEL quadrupoles. Analogous results are found for the vertical offset, where the sag correction must be applied [6].

TABLE I
SETUP CHARACTERISTICS AND LOCK-IN AMPLIFIER PARAMETERS

Wire and magnet characteristics	
Wire current amplitude	75 mA
Second harmonic frequency	202.55 Hz
Quadrupole strength at 10 A	0.36 T
Quadrupole effective length	170 mm
Lock-in amplifier filter settings	
Time constant	140 ms
dB/Oct	24
Bandwidth	490 mHz
Lock-in amplifier PLL settings	
Time constant	4.45 ms
dB/Oct	24
Bandwidth	16 Hz
Proportional gain	34.76 mHz/degree
Time constant	71.2 ms

TABLE II
ANALYSIS OF THE AXIS MEASUREMENT OF FIG. 4

	σ_{meas} [mV]	σ_{fit} [mV]	S [mV/µm]	Inters. [µm] Value [mV]	R [µm]
		Without PLL			
$x < x_0$	0.2	5.5	-0.74	-0.4	1.0
$x > x_0$	0.6	7.8	0.72	0.2	1.9
		With PLL			
$x < x_0$	0.1	0.7	-0.75	0.0	0.2
$x > x_0$	0.1	0.6	0.75	-0.1	0.2

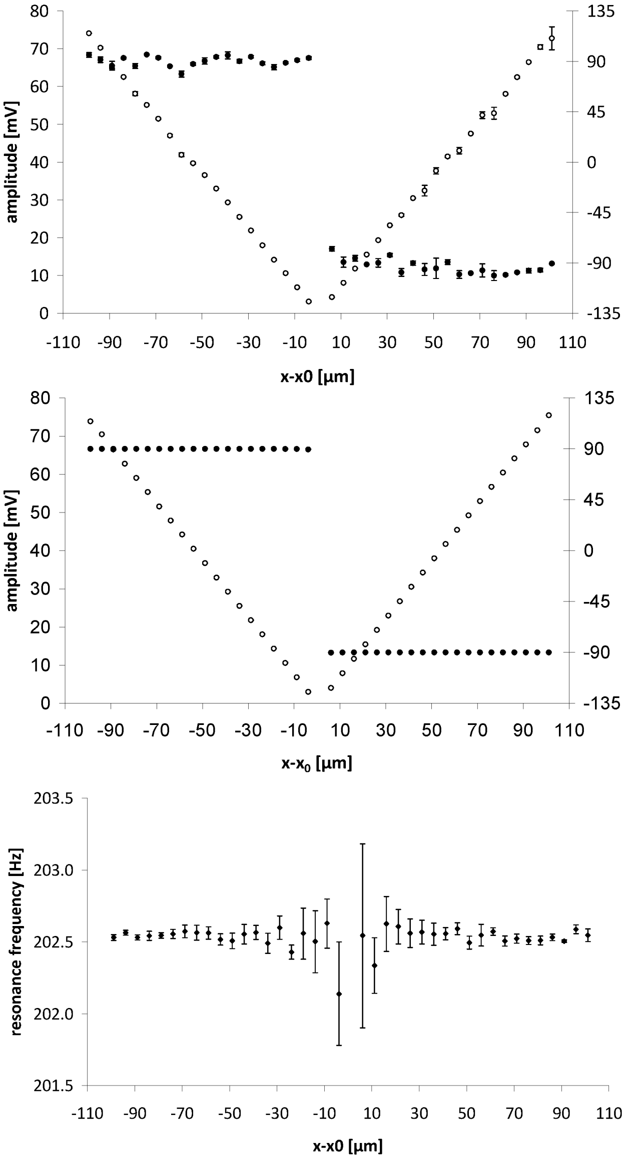

Fig. 4. Quadrupole axis offset measurement without PLL (upper figure, amplitude: black dots, phase: black circles) and with PLL (middle figure). The stability of the resonance frequency for each wire offset for the measurements with PLL is shown in the bottom figure.

B. *Quadrupole axis pitch & yaw measurements*

The angle between the quadrupole geometrical axis and the magnetic axis projected in the *YZ*-plane and *ZX*-plane is respectively, the pitch and yaw. The change in resonance frequency due to wire movement when measuring the pitch and yaw angles, has to be taken into account and corrected for. The resonance frequency of the stretched wire at length L is given by (see Eq. 2)

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \tag{4}$$

When the wire is further stretched by moving one end as it would be the case during pitch/yaw measurements, its natural frequency will change due to the wire's change in tension and length. The frequency change can be estimated from Young's elasticity model:

$$\Delta T = \frac{EA_0 \Delta L}{L} \tag{5}$$

This states that the applied force to the wire, the tension, is proportional to its initial cross section A_0, initial length L, length change ΔL, and Young's constant E. Moving one end of the wire results in an elongated wire with length $L+\Delta L$ and tension $T+\Delta T$ and hence, a modification in natural wire frequency (neglecting changes in the wire cross section)

$$\Delta f_n = \frac{n}{2(L+\Delta L)} \sqrt{\frac{T+\Delta T}{\rho A_0}} - f_n \tag{6}$$

Inserting T from (4) this reduces to

$$\Delta f_n = \frac{n}{2(L+\Delta L)} \sqrt{\frac{(L+\Delta L)(4 f_1^2 L^3 \rho + \Delta L E)}{L^2 \rho}} - f_n \tag{7}$$

For the copper beryllium wire (ρ=8.25·10^3 kg/m^3, E=140·10^9 N/m^2) stretched to such tension that f_1=100 Hz (Eq. 4) and clamped at L=1.2 m, the change in resonance frequency when moving one end of the wire by 1 mm (ΔL=0.5 µm) will be ~10 mHz. Since the pitch and yaw measurements will be carried out in the fourth harmonic, the expected change in frequency is of the order of 40 mHz. Despite being small, it will affect the measurement results. Also, extreme care should be taken to place the stages that move the wire parallel to each other. If the stages have an angle of 1 mrad, according to (Eq.7) this will cause a change in resonance frequency in the second harmonic of 25 mHz per mm-wire movement. An accurate positioning of the stages was achieved using the FARO arm.

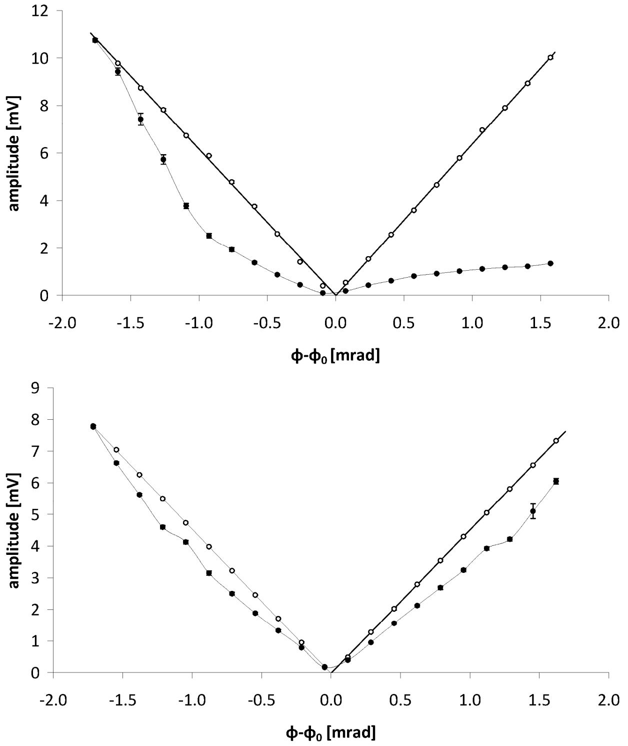

Fig. 5. Yaw (upper figure) and pitch (bottom figure) angle measurement without PLL (black dots) and with PLL (black circles, linear fit).

The yaw and pitch angles have been measured with and without PLL by recording the vibration signals at different wire angles. The measured amplitudes with RMS error values are shown in Fig. 5 for each relative wire angle Φ-Φ_0 (Φ_0 being the calculated angle from the two linear fits). The analysis is summarized in Table III. Using PLL the yaw and pitch angles are well defined with an error less than 0.02 mrad. In the measurements where the resonance frequency was not corrected for, no proper linear fit can be made and the error in finding the pitch and yaw angles is of about 0.1 mrad.

TABLE III

ANALYSIS OF THE AXIS MEASUREMENT OF FIG. 5

	σ_{meas} [µV]	σ_{fit} [µV]	S [µV/µrad]	Inters. [mrad] Value [µV]	R [µrad]
			Yaw, without PLL		
$\Phi < \Phi_0$	104.9	888.5	-7.11	0.4	140
$\Phi > \Phi_0$	16.2	83.6	0.72	8.9	138
			Yaw, with PLL		
$\Phi < \Phi_0$	34.8	99.6	-6.16	0.0	22
$\Phi > \Phi_0$	34.0	51.6	6.36	11.0	13
			Pitch, without PLL		
$\Phi < \Phi_0$	42.2	272.0	-4.57	-0.1	69
$\Phi > \Phi_0$	63.5	147.3	3.60	-308.8	58
			Pitch, with PLL		
$\Phi < \Phi_0$	18	10.4	-4.56	0.0	6
$\Phi > \Phi_0$	17	5.8	4.53	-20.3	5

IV. CONCLUSION

The advantage of a PLL is that it ensures the maintaining of the resonance condition even when the wire resonance is disturbed. In measurement processes where the wire instead of the magnet is moved to find the magnetic axis, the use of this control system is particularly suited. For quadrupole pitch and yaw measurements, an increased signal stability and improved reliability of measurement outcomes have been demonstrated.

V. ACKNOWLEDGMENT

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